#### <u>Prob. 1</u>

A material with two energy levels and photons are at the equilibrium state as shown below. The photon energy,  $E_p$ , is equal to  $E_2-E_1=100meV$ . The rate of spontaneous emission can be represented as  $AN_2$  and that of absorption as  $BN_1N_p$ , where  $N_1$  is the density of electrons at  $E_1$ ,  $N_2$  is the density of electrons at  $E_2$ ,  $N_p$  is the photon density, and A, B are constants. Note that the material has no holes. Use kT=25meV.

(a)(10) What is the expression for the stimulated emission rate?

(b)(10) Determine the numerical value of  $N_1/N_2$ , the ratio between electron densities at  $E_2$  and  $E_1$ . (Hint: Photon density at equilibrium is proportional to 1/ ( $e^{E/kT}-1$ ), where E is the photon energy.)

(c)(10) What is the percentage of photons that are due to stimulated emission?

(d)(10) Electron are excited from  $E_1$  to  $E_2$  by optical pumping. If the total density of electrons in the material is N, what should be  $N_2$  in order to reach the transparency condition?



### Prob. 2

Consider a laser made up of a gain material and two external mirrors as shown below. The external mirrors have reflectivity of 0.3. The end facets of the gain material have anti-reflection coatings so that their reflectivities are zero. The gain of the material is a function of wavelength and injected carrier density:  $g(\lambda,n) = a (n-n_0) - b (\lambda-\lambda_0)^2$ , where,  $a = 2.4 \times 10^{-17} \text{ cm}^2$ ,  $n_0 = 1 \times 10^{18} / \text{ cm}^3$ , and  $b = 4800 / \text{ cm} - \mu \text{m}^2$ , and  $\lambda_0 = 1.0 \mu \text{m}$ . The reflective index of the material is 3 and there is no internal loss. Use  $\Gamma=1$ .

(a)(10) Determine the resonance condition for the lasing wavelength?

(b)(10) At what wavelength in µm does the first lasing mode appear?

(c)(10) What is the threshold gain in  $cm^{-1}$  for the first lasing mode?

(d)(10) An optical amplifier is made by removing two mirrors. If the injected carrier density is twice of the threshold carrier density, what is the output optical power for input light at 1.0 µm?



## <u>Prob. 3</u>

(a)(10) What is the external quantum efficiency of a laser? Why is it less than in a practical laser?

(b)(10) What is a DFB laser?

### <u>Prob. 4</u>

Consider a laser made up of a gain material and two external mirrors as shown below. The external mirrors have reflectivity of 0.3. The end facets of the gain material have anti-reflection coatings so that their reflectivities are zero. The power gain of the material is a function of wavelength and injected carrier density :  $g(\lambda,n) = a (n-n_0) - b (\lambda - \lambda_0)^2$ , where,  $a = 8 \times 10^{-16} \text{ cm}^2$ ,  $n_0 = 1 \times 10^{18} / \text{cm}^3$ , and  $b = 4800 / \text{cm} - \mu \text{m}^2$ , and  $\lambda_0 = 1.0 \mu \text{m}$ . The reflective index of the material is 3 and there is no internal loss. Use  $\Gamma = 1$ .  $l_1 = l_2 = l_3 = 10 \mu \text{m}$ .

(a)(10) What is the threshold carrier density for lasing?

(b)(10) As more carriers are injected than determined in (a), the laser becomes a multimode laser. What wavelength does the second dominant mode lase at?

(c)(10) We want to replace mirrors so that the minimum injected carrier density for lasing is reduced by 50%. What is the required mirror reflectivity? Assume two identical mirrors are used.



## <u>Prob. 5</u>

An APD has gain of 20 and responsivity of 12 A/W at  $1.55 \mu m$  .

(a)(5) What is the quantum efficiency of the detector? (Remember the quantum efficiency is less than 1)

(b)(5) If 10<sup>10</sup> photons are incident on the APD for one second, how much photocurrent is generated?

(c)(10) If the dominant noise for the detector is the shot noise and the gain process is noise-free, what is the SNR of the detector in dB?

(d)(10) The APD detector is used in a 10Gbps optical communication system. The system requirement sets the minimum allowed SNR to be 1dB. Assuming again the shot noise is dominant, determine the minimum optical power that should be incident on the detector.

### <u>Prob. 6</u>

(a)(5) Why does a semiconductor laser with cleaved mirrors have many lasing modes?

(b)(5) Why is the diret modulation of semiconductor lasers not desirable for very high speed optical communication systems?

(c)(5) What distinguishes LED, semiconductor optical amplifier, and semiconductor laser?

(d)(5) How can a photoconductor have gain?

Consider a semiconductor whose band structure is shown below. The bandgap is  $E_g$  and the conduction band effective mass,  $m_e^*$ , and the valence band effective mass,  $m_h^*$ , are constant and  $m_h^* = 2 m_e^*$ .



(a)(10) Carriers are injected into the semiconductor until the semiconductor is transparent for photons whose energy is Eg+ $\Delta$ E. What is the separation of quasi-Fermi energies in this semiconductor? Assume the bandgap does not change with the injected carriers.

(b)(15) What is the conduction-band quasi-Fermi energy for the situation given in (a)? The electron density of states is given as  $g_0(m_e^*)^{3/2}(E-E_g)^{1/2}$  and the semiconductor is at 0K.

(c)(10) An optical amplifier is made with the semiconductor having length of 0.5cm. In order to make the amplifier have 20dB power gain at  $\lambda = 1 \mu m$ , what is the required injected carrier densities? Assume the amplifier has end facets with zero reflectivities, g the gain coefficient, at  $\lambda = 1 \mu m$  is given as a(N-N<sub>0</sub>) for where a =  $10^{-17} \text{ cm}^{-2}$ , N<sub>0</sub> is  $10^{18} \text{ cm}^{-3}$ , and  $\Gamma = 0.1$ .



(d)(10) The above amplifier is converted into a laser by making both end facets to have non-zero reflectivity R. If the lasing threshold carrier density is  $2N_0$ , what is the mirror reflectivity? Assume there is no internal loss.

(e)(10) The laser is observed to have ten lasing modes at around  $\lambda=1\mu m$ . What is an

estimate in nm for the range in which the semiconductor has optical gain? Assume the refractive index of the semiconductor is 3.5.

(f)(10) We want to make a single mode laser by reducing the cavity length of the laser in (e). Determine the minimum cavity length so that the laser has a single mode. Assume all other conditions are the same as in (e).

#### <u>Prob. 8</u>

(a)(10) An intrinsic semiconductor has the electron and hole concentration of  $10^{10}$  cm<sup>-3</sup> and the recombination time of  $10\mu$ s. A photoconductor is made of this semiconductor. If  $1 \text{mW/cm}^3$  photon power density at  $\lambda$ =1 $\mu$ m is incident at the photoconductor, how many carriers are produced at the steady state? Assume the photoconductor has the internal quantum efficiency of 0.5.

(b)(10) How much does the resistance of the photoconductor increase (or decrease)? Assume the electron and hole mobilities are identical.

(c)(15) An APD has separate absorption and gain regions and the gain region has k  $(= \alpha_h/\alpha_e)$  of 1 and length W. Derive the APD gain.

### <u>Prob. 9</u>

A ring laser has a gain material inside a cavity made of two perfectly-conducting metallic mirrors and one mirror with power reflectivity R=0.5 as shown below. Assume there is no reflection at the end facets of the gain material. Assume the laser has no internal loss.

(a)(10) How much threshold power gain should the gain material have in order for the laser to lase?

(b)(10) The output spectrum of the laser has multi-modes. What is the mode separation in Hz?

(c)(10) The reflective index of the gain material has dependence on lightwave frequency around  $\omega_0$ :  $n(\omega) = n_0 + n' (\omega - \omega_0)$  with positive n'. Does the mode separation in Hz between multi-modes around  $\omega_0$  decrease or increase? Explain why.





A PIN photodetector has responsivity of 0.5A/W at 1.55 $\mu m$  .

(a)(10) What is the quantum efficiency of the detector?

(b)(10) In order to produce 1nA of detection current, how many photons should be incident on the detector per second?

(c)(10) If the dominant noise for the detector is the shot noise, what is the SNR of the detector in dB?

(d)(10) The detector is used in a 10Gbps optical communication system. The system requirement sets the minimum SNR to be 1dB. Assuming the shot noise is dominant, determine the minimum optical power that should be incident on the detector.

### Prob. 11

A photon has energy of 1 eV. What is the wavelength in um for this photon? Use q=1.6 x  $10^{-19}$  C and h=6.625 x  $10^{-34}$  Jsec.

## Prob. 12

A semiconductor laser has following properties.

- Cavity length: 500 μm	– Active region thickness: 0.2 µm
- Active region width: 2 μm	- Confinement factor: 0.15
- Internal loss: 6 cm <sup>-1</sup>	- Mirror reflectivity (both facets): 0.3

#### - Effective index: 3.5 - Carrier lifetime at threshold: 2 nsec

The gain characteristics for the active region material are shown in the figure shown below.



Answer the following questions. Use interpolation when necessary.

(a)(10) What is the threshold gain in  $cm^{-1}$  for the laser?

(b)(10) At what wavelength can the first lasing mode be observed?

(c)(10) Estimate the threshold current for the first lasing mode.

(d)(10) As the injected current increases, more than one lasing modes are observed. What is the mode separation in nm?

(e)(10) With the injected carrier density of  $1.8 \times 10^{18}$  cm<sup>-3</sup>, how many lasing modes are there? (f)(10) It is found that the refractive index for the active region and the cladding layers increases with increasing  $\lambda$ . Does the lasing mode separation increase or decrease? Justify your answer.

The above laser diode can be converted to an optical amplifier if two facets are antireflection coated. For simplicity in numerical calculation, assume for the following problems that the internal loss of the amplifier is  $50 \text{ cm}^{-1}$ , and the confinement factor is 0.2.

(g)(10) At the injected carrier density of  $1.8 \times 10^{18}$  cm<sup>-3</sup>, what is the wavelength bandwidth in nm for the optical amplifier?

(h)(10) At the injected carrier density of  $1.8 \times 10^{18} \text{ cm}^{-3}$ , what is the largest signal gain ( $I_{out}/I_{in}$ ) that can be achieved with this optical amplifier?

#### Prob. 13

A careful observation of a laser shows that lasing spectrum is not a delta function but has a certain linewidth. Based on the things that we learned in class, what can be possible causes for this?

Consider a simple optical fiber link which consists of a semiconductor laser transmitter, fiber, and a PIN receiver as shown below. The laser has a single mode lasing wavelength at 1.55  $\mu$ m, the threshold current of 10 mA and the external quantum efficiency of 0.5. Assume the laser has only one output facet. The fiber has transmission power loss of 0.3dB/km. The PIN PD has the (external) quantum efficiency of 0.8. Assume there are no coupling losses between LD and fiber, and fiber and PD (all the powers from LD is coupled into fiber and from fiber to PD). Assume the optical link operates at room temperature, all the noises are generated at the receiver and the application for this link requires the bandwidth of 1GHz. Use KT/q = 25 mV at room temperature.



(a)(10) How much optical power comes out of the laser if the laser driver current I =15mA?

(b)(10) If the fiber length is 100Km, how much currents are produced at the receiver?

(c)(10) Is the system shot-noise limited or thermal-noise limited?

(d)(10) What is the signal-to-noise ratio (SNR) for the case in (b)?

(e)(10) If the PIN PD is replaced with an APD with gain of 10 and excess noise figure of 1.5, what is the SNR of the system? Assume the APD has the same internal quantum efficiency as the PIN PD.

### Prob. 15

We can experimentally determine internal quantum efficiency,  $\eta_{int}$ , and internal loss,  $\alpha_{int}$ , in semiconductor lasers by measuring the slope of L-I curves (i.e. dP/dI) for several lasers fabricated under the identical conditions but having

different cavity lengths. Assume the laser has two identical cleaved mirrors with reflectivity R.

- (a)(5) Express the external quantum efficiency,  $\eta_{ext}$ , as function of dP/dl.
- (b)(5) What is the relationship between  $\eta_{ext}$  and  $\eta_{int}$ ?
- (c)(5) Using the result of (b), devise a method for determining  $\eta_{\rm int}$  and  $\alpha_{\rm int}$ .

### Prob. 16

A semiconductor laser with a shortest possible cavity length L is realized with a simple PIN structure and dielectric mirrors with high reflectivity as shown below. The laser lases at 1um and the waveguide has effective index of 3 at 1um and confinement factor of 0.1 Assume the PIN structure has no internal loss.



(a)(10) What is the shortest possible cavity length L?

(b)(10) The laser has threshold gain of 10000 cm-1. Assuming the gain in the laser is uniform, determine reflection coefficient r2?

(c)(10) In average, how many round-trips between two mirrors do the photons in the laser make before they escape the laser? Approximate the photon group velocity with its phase velocity.

(d)(10) Sketch the photon distribution along the cavlity (z-axis).

## <u>Prob. 17</u>

Determine whether each of the following statements is True or False. Briefly but clearly explain why. Without a clear and correct explanation, you will receive no credit even if your T/F answer is correct.

(a)(10)In general, lasers having larger volumes require less amount of pumping power for lasing.

(b)(10) In general, it is more difficult to realize lasers lasing at shorter wavelength than longer wavelength.

(c)(10) In optical amplifiers, amplification gain becomes smaller under the same amount of pumping power as the input optical signal power becomes larger.

## Prob. 18

We want to design a circular VCSEL (Vertical Cavity Surface Emitting Laser) lasing at 1um whose structure is shown below. The values for important laser and material parameters are also given. For simplicity, assume there is no internal loss, the optical confinement factor is 1, the refractive indices for both active region and claddings are 3 ( $n_1 = 3$ ), and the bottom mirror has the reflectivity of 1.



(a)(5) Determine the minimum possible value for L, the laser cavity length. (b)(10) We want the VCSEL to have the threshold current of 1mA. What is  $n_{th}$ , the threshold carrier density in cm<sup>-3</sup>, and  $g_{th}$ , threshold gain in cm<sup>-1</sup>? (c)(10) What is the top mirror reflectivity in order to realize (b)? (d)(5) The top mirror can be realized by stacking up two materials: one with  $n_2 = 2.2$  and the other with  $n_3 = 1.1$ . What is the layer thickness for  $n_2$  and  $n_3$ ?

(e)(5) Which layer should be stacked first, layer with  $n_2$  or  $n_3$ ? Why? Assume the active region is in the middle of the laser cavity.

(f)(5) What is the minimum number of staked layers required? Assume the laser is located in the vacuum.

(g)(10) It is experimentally found that the designed VCSEL has multi-mode characteristics. Explain why this is the case and propose possible ways of making the VCSEL a single-mode laser.

## Prob. 19

Answer the following questions.

(a)(5) Why does responsivity of a photodector depend on the incident light wavelength?

(b)(5) Why does a photoconductor has a gain?

(c)(5) What are **the** possible ways to enhance the gain of a photoconductor? Explain.

(d)(5) Why is the intrinsic region required in a practical PIN photodiode?

# Prob. 20

A fiber ring laser lasing around 1.55µm is realized with a piece of Er-doped fiber (EDF) and a 3-dB beam splitter as shown below. The 3-dB beam splitter divides the input power into two equal output powers. Assume all the pump power transmitted by the beam splitter is absorbed by EDF and the resulting excited carriers are uniformly distributed within EDF. Also assume the reflected pump power is filtered out by an optical filter so that only the laser output is present at the output. Values of parameters that are needed to

solve this problem are given below.



- $\Gamma(EDF \text{ confinement factor}): 0.1$
- l (EDF length): 1m

g (gain in EDF) = 
$$a(N - N_0)$$
  
where  $a=10^{-24}$ m<sup>-2</sup>,  $N_0=10^{25}$ m<sup>-3</sup>

- $\eta_i$  (internal quantum efficiency in EDF): 1
- $\alpha_{\rm int}$  (internal loss in EDF): 0
- $\tau$  (carrier lifetime in EDF): 1 msec

$$V = 10 - 10 \text{m}3$$

$$\lambda_{\text{pump}} = 0.98 \ \mu \text{m}$$

- (a)(10) What is the threshold gain of the laser in 1/m?
- (b)(10) What is the excited carrier density at the threshold in  $1/m^3$ ?
- (c)(10) What is the threshold pump power (P<sub>pump</sub>) required for lasing in mW?
- (d)(10) The laser produces multi-mode lasing spectrum. What is the mode separation in wavelength at around 1.55µm?
- (e)(10) What is the external quantum efficiency of the laser?

## <u>Prob. 21</u>

A simple optical fiber link is realized with a DFB transmitter, fiber, and a PIN PD receiver as shown below. Values of parameters that are needed to solve this problem are given below.



(a)(10) It is observed that the noises in the receiver have same contributions from thermal noises and shot noises. What are the currents in the receiver?(b)(10) What is the bias current for the DFB laser in order to produce the currents obtained in (a).

(c)(10) What is the probability that a given injected carrier into the laser in the transmitter side causes the current flow in the receiver?

(d)(10) It is observed that even with the DFB laser which has a single lasing mode, optical pulses sent from the transmitter experience a significant amount of broadening (dispersion) when they reach the receiver side. What in the DFB laser can cause this (Hint: When the laser output power changes, the carriers inside laser change as well)?

# <u>Prob. 22</u>

Both DFB (Distributed FeedBack) laser and VCSEL (Vertical Cavity Surface Emitting Laser) can produce single mode lasing spectrum. Discuss advantages and disadvantages of VCSEL over DFB. (Give at least one advantage and one disadvantage)

# Prob. 23

Carriers are pumped into the active region in a double heterojunction PIN structure shown below. Use parameter values given below for answering following questions.



(a)(10) When the structure is lasing, photons can escape from the structure through two mirrors. What is the average time in seconds for a photon to escape?

(b)(10) Assume photons are traveling inside the active region having optical gain coefficient g. Their number increases due to optical gain. Determine the expression for the rate of this increases over time at any given location, or  $dN_p(z)/dt$ . [Hint: Use  $N_p(z+\Delta z) = N_p(z)+\Delta N_p = N_p(z)exp(g\Delta z) = N_p(z)(1+g\Delta z)$ ].

(c)(10) Determine the threshold gain,  $g_{th}$ , required for lasing.

The same PIN structure described in Prob. 1 can be used as an optical amplifier. Assume input wavelength is around  $\lambda=1\mu m$  and the structure is not lasing since g is less than  $g_{th}$ .

(a)(10) It is observed that the signal gain (G= P<sub>out</sub>/P<sub>in</sub>) have several peaks as schematically shown below. Explain why such peaks exist and determine the separation of peaks in nm



The signal gain peaks can be eliminated by coating a dielectric layer on each mirror so that R<sub>1</sub>=R<sub>2</sub>=0 as shown below.



(b)(10) Determine refractive index and thickness (in  $\mu$ m) of the dielectric layer.

(c)(10) Now the carrier pumping is increased so that the optical gain coefficient of the active region is  $1000 \text{ cm}^{-1}$  for  $\lambda = 1 \mu \text{m}$ . What is the signal gain (P<sub>out</sub>/P<sub>in</sub>) in dB at  $\lambda = 1 \mu \text{m}$ ?

The same PIN structure shown in Prob. 1 and 2 with  $R_1=0.25$  and  $R_2=0$  can be used as a photodetector when biased reversely as shown below. Experiments with this photodetector show when 1mW light at  $\lambda=1\mu m$  is shined as input ( $P_{in}$ ), 0.1mW is produced as output light ( $P_{out}$ ).



(a)(10) Assuming all the absorbed photons are converted into electric carriers in photo-generated currents, what is the photo-generated current? [Hint: Do not ignore the reflection at R<sub>1</sub>.]

(b)(10) We want to make sure that there is virtually no output light. What changes in the PIN structure can achieve this? Give at least two possibilities along with explanations.

(c)(10) Determine the SNR in dB for the photo-generated currents when the measurement is taken for 1nsec. Consider only the shot noise.